Sequential optimal control of the nuclear fuel reload mechanism

Cite as: AIP Conference Proceedings **2172**, 070015 (2019); https://doi.org/10.1063/1.5133551
Published Online: 13 November 2019

Yu. F. Dolgii, A. N. Sesekin, O. L. Tashlykov, and K. T. Tran

Sequential Optimal Control of the Nuclear Fuel Reload Mechanism

Yu. F. Dolgii[1,2], A. N. Sesekin[1,2,a)], O. L. Tashlykov[1] and K. T. Tran[1]

[1]*Ural Federal University, 19 S. Mira, Ekaterinburg, 620002, Russia.*
[2]*N. N. Krasovskii Institute of Mathematics and Mechanics, UB of RAS, 16 S. Kovalevskaya, Ekaterinburg, 620990, Russia.*

[a)]Corresponding author: sesekin@list.ru

Abstract. The BN-800 reactor overload system is designed to overload fuel assemblies and consists of a set of nodes that provide guidance for the reload mechanism at given coordinates, grabbing, lifting, lowering and rotating assemblies. On the throat of the BN-800 reactor there are three rotary plugs, the smaller of which is located inside the middle one and the middle one inside the large one. On a smaller tube placed the capture mechanism of the fuel assembly. The plugs, which are the reactor cover, perform the role of thermal and biological protection, as well as, guide the reload mechanism to the given coordinates of the core in order to capture the fuel assembly and move it to the required zone with the given coordinates. In this paper, we will consider the problem of minimal-time cast a gripper located on a smaller traffic jam to a given fuel assembly, assuming that the traffic jams will turn in series and at each moment only one traffic jam can be turned. The solution of such a task will contribute to the reduction of the stopping time of the power unit for carrying out operations on refueling. For this problem, a mathematical model was constructed to describe the movement of three connected plugs. Based on that, an algorithm for constructing optimal control was proposed considering given assumptions.

INTRODUCTION

Nuclear power plants generate electricity due to the heat generated during the fission chain reaction flowing in the core of a nuclear reactor. Uranium dioxide fuel pellets containing a fissile isotope in an airtight envelope represent a fuel element. Fuel elements have a small diameter (for a WWER-1000 reactor - 9.1 mm; BN-600 and BN-800 - 6.9 mm) and a length of several meters; therefore, for convenience of transportation and protection against mechanical damage, they are combined in number from several dozens to several hundred pieces in a single rigid structure - a fuel assembly (FA). A nuclear rector can work only if there is a certain amount of fuel in the core, which corresponds to a critical mass. The reactor operation nuclear fuel gradually burns out, the number of fissioning nuclei in it decreases. At the same time, fission products are formed, thereby adversely affecting the fission process in the reactor. Therefore, after a certain period, called the campaign of fuel (3-4 years), fuel assemblies must be unloaded from the reactor. Then the fresh fuel assemblies are install in their place. The main goal of the nuclear power plant is to ensure the generation of electricity in a safe and cost-effective way. The main ways to improve the efficiency of electricity generation are to reduce the shutdown time of nuclear power plants for nuclear fuel reloading and equipment repair. In WWER-type reactors, the capture of the transshipment machine to the required coordinates is induced when it is moved by the bridge and the truck carriage in two mutually perpendicular coordinates. In order to unload the burned-out fuel assembly from the reactor, the transshipment vehicle must move to its coordinates, lower the grip, engage the gear, raise the grip from the fuel assembly, transfer it to the swimming pool, lower it, disengage and lift the grip. These operations are performed for each FA that is being unloaded, regardless of the overload order. In the reverse order the fresh fuel assembly is loaded. The time spent on the operation of lowering-raising the capture of the overloading machine, the coupling-disengagement of the capture from the fuel assembly can be ignored in the calculation algorithm, since it is a parameter that cannot be optimized in this context. As a result, the task can be presented as flat, with some of the movements not being considered [1, 2]. The considered task of optimizing the sequence of operations for rearranging fuel assemblies consists in minimizing the time for replacing nuclear

Proceedings of the 45th International Conference on Application of Mathematics in Engineering and Economics (AMEE'19)
AIP Conf. Proc. 2172, 070015-1–070015-6; https://doi.org/10.1063/1.5133551
Published by AIP Publishing. 978-0-7354-1919-3/$30.00

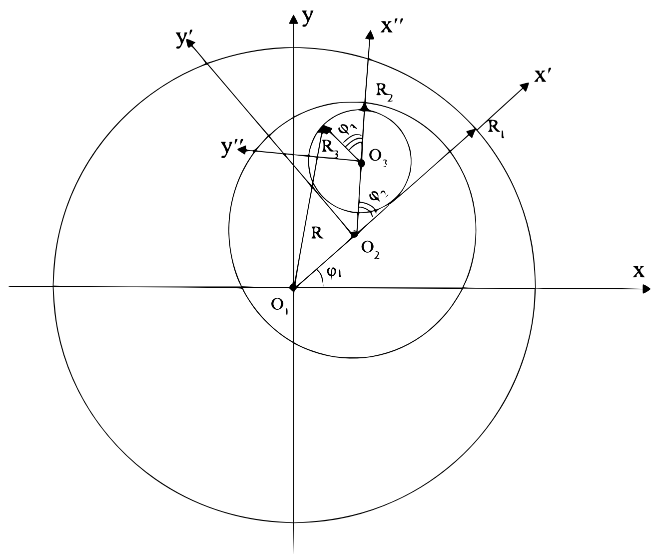

FIGURE 1. Scheme of rotary plugs.

fuel and, accordingly, reducing the downtime of the NPP unit. The technological feature of fast-neutron reactors with sodium coolant is the guidance of the overload mechanism to the required coordinates of the fuel assembly by rotating eccentrically arranged devices (turning plugs). The BN-800 reactor overload system is designed to overload fuel assemblies and consists of a set of nodes that provide guidance for the overload mechanism at given coordinates, grabbing, lifting, lowering and rotating assemblies. On the throat of the BN-800 reactor there are three rotary plugs, the smaller of which is located inside the middle one and the middle one inside the large one. The capture mechanism of the fuel assembly is placed on a smaller plug. The plugs, which are the reactor cover, perform the role of thermal and biological protection, as well as, guiding the overload mechanism to the given coordinates of the core in order to capture the fuel assembly and move it to the required zone with the given coordinates. In this paper, we will consider the task of steadily targeting a grip located on a smaller plug to a given fuel assembly, assuming that the plugs will rotate sequentially, and only one tube is possible to rotate at each moment. The solution of such a task will contribute to the reduction of the stopping time of the power unit for carrying out operations on refueling. For this task, a mathematical model was constructed that describes the movement of three connected plugs. Based on it, an algorithm for constructing optimal control is proposed under the assumptions made.

MATHEMATICAL MODEL DESCRIBING THE PLUGS DYNAMICS

The mechanical system consists of three turning plugs. A large cork is a disk of radius R_1, the geometric center of which remains motionless during movement. The large cork has an eccentric circular notch of radius R_2, inside of which an average cork is placed, which is a disk of radius R_2, the geometrical center of O_2 of which remains motionless while moving relative to a large cork. The middle plug has an eccentric circular cutout of radius R_3. Inside this cutout there is a small cork, which is a disk of radius R_3, the geometric center O3 of which, while moving, remains stationary relative to the average cork (see Fig. 1).

We neglect the forces of friction when describing the interaction of traffic plugs. To describe the dynamics of the considered mechanical system with three degrees of freedom, we will use the Lagrange equations of the 2nd kind [3]. As the generalized coordinates, we choose the angle of rotation of the large plugs φ_1, the angle of rotation of the middle plugs φ_2 with respect to the large plugs and the angle of rotation of the small plugs φ_3 relative to the center plugs. The control moments u_1, u_2 and u_3, which are applied to the large, medium and small plugs traffic, respectively, are generalized forces.

The kinetic energy of this mechanical system without additional kinematic restrictions is determined by the formula

$$T = \frac{1}{2}J_1\dot\varphi_1^2 + \frac{1}{2}J_2\left(\dot\varphi_1 + \dot\varphi_2\right)^2 + \frac{1}{2}J_3\left(\dot\varphi_1 + \dot\varphi_2 + \dot\varphi_3\right)^2 + \frac{1}{2}m_2\left(e_2^2\dot\varphi_1^2 + 2e_2 a\cos(\varphi_2 + \alpha)\dot\varphi_1(\dot\varphi_1 + \dot\varphi_2)\right)$$

$$+\frac{1}{2}m_3\left(e_2^2\dot\varphi_1^2 + e_3^2(\dot\varphi_1 + \dot\varphi_2)^2 + 2e_2e_3\cos(\varphi_2)\dot\varphi_1(\dot\varphi_1 + \dot\varphi_2)\right).$$

Here J_1, J_2 and J_3 are the moments of inertia of the large, medium plugs with notches and the small plugs relative to the O_1, O_1, and O_1, axes, respectively, m_2 is the mass of the average plug with a notch, m_3 is the mass of the small plug, $e_2 = |O_1O_2|$, $e_3 = |O_2O_3|$, $a = |O_2C_2|$, $\alpha = \angle(O_3O_2C_2)$, C_2 is the center of mass of the middle plug with a notch, and O_3 is the center of mass of the small plug.

The mathematical model of the transshipment device is described using the following Lagrange equations of the second kind

$$J_1\ddot\varphi_1 + J_2\left(\ddot\varphi_1 + \ddot\varphi_2\right) + J_3\left(\ddot\varphi_1 + \ddot\varphi_2 + \ddot\varphi_3\right) + m_2\left(e_2^2\ddot\varphi_1 + e_2 a\cos(\varphi_2 + \alpha)(2\ddot\varphi_1 + \ddot\varphi_2) - e_2 a\sin(\varphi_2 + \alpha)(2\dot\varphi_1 + \dot\varphi_2)\dot\varphi_2\right)$$

$$+m_3\left(e_2^2\ddot\varphi_1 + e_3^2(\ddot\varphi_1 + \ddot\varphi_2) + e_2e_3\cos(\varphi_2)(2\ddot\varphi_1 + \ddot\varphi_2)\ -e_2e_3\sin(\varphi_2)(2\dot\varphi_1 + \dot\varphi_2)\dot\varphi_2\right) = u_1,$$

$$J_2\left(\ddot\varphi_1 + \ddot\varphi_2\right) + J_3\left(\ddot\varphi_1 + \ddot\varphi_2 + \ddot\varphi_3\right) + m_2e_2a\left(\cos(\varphi_2 + \alpha)\ddot\varphi_1 + \sin(\varphi_2 + \alpha)\dot\varphi_1^2\right)$$

$$+m_3\left(e_3^2(\ddot\varphi_1 + \ddot\varphi_2) + e_2e_3(\cos(\varphi_2)\ddot\varphi_1 + \sin(\varphi_2)\dot\varphi_1^2)\right) = u_2,$$

$$J_3\left(\ddot\varphi_1 + \ddot\varphi_2 + \ddot\varphi_3\right) = u_3.$$

As a result, we got a complex control system. We will simplify the control problem. We will assume that a complex rotation of a mechanical system, at which the angles φ_1, φ_2, and φ_3 simultaneously change, will be replaced by three successive rotations of the plugs.

KINETIC EQUATIONS

In this section, we will establish the connection between the Cartesian coordinates of the point (x, y) (vector $\vec{R}$), which contains the fuel assembly capture mechanism, and the three angles that determine the position of the point (x, y). To find the coordinates of the vector $\vec{R}$, we introduce two new coordinate systems $O_2x'y'$ and $O_3x''y''$, the beginnings of which correspond to the geometric centers of the second and third traffic plugs, respectively, and these systems are fixedly connected with the corresponding plugs. The O_2 point has coordinates $(l_2\cos\varphi_1, \sin\varphi_1)$ in a fixed coordinate system Oxy associated with the center of the first plugs. Considering the rotation of the coordinate system $O_2x'y'$ with respect to the coordinate system O_1xy by the angle φ_1 and a shift by the value of l_2 in the direction of the axis O_1O_2, we get the following equalities:

$$\left\{\begin{array}{lcl} x &=& x'\cos\varphi_1 - \sin\varphi_1 + l_2\cos\varphi_1 = (x' + l_2)\cos\varphi_1 - y'\sin\varphi_1 \\ y &=& x'\sin\varphi_1 + \cos\varphi_1 + l_2\sin\varphi_1 = (x' + l_2)\sin\varphi_1 + y'\cos\varphi_1. \end{array}\right. \tag{1}$$

The coordinates in the $O_3x''y''$ and $O_2x'y'$ systems are similarly connected, where one of the systems is rotated relative to the other by the angle φ_2 and shifted by the value of l_3 in the direction of the axis O_2x:

$$\left\{\begin{array}{lcl} x' &=& (x'' + l_3)\cos\varphi_2 - y''\sin\varphi_2 \\ y' &=& (x'' + l_3)\sin\varphi_2 + y''\cos\varphi_2. \end{array}\right. \tag{2}$$

The coordinates of the point (x, y) in the coordinate system, fixedly connected with the small plug, have the form

$$\left\{\begin{array}{lcl} x'' &=& R_3\cos\varphi_3 \\ y'' &=& R_3\sin\varphi_3. \end{array}\right. \tag{3}$$

In order to get a representation of the coordinates of a point (x, y) through three angles, firstly we substitute (x'', y'') from (3) into system (2). After applying the known trigonometric formulas, we'll get

$$\left\{\begin{array}{lcl} x' &=& l_3\cos\varphi_2 + R_3\cos(\varphi_2 + \varphi_3) \\ y' &=& l_3\sin\varphi_2 + R_3\sin(\varphi_2 + \varphi_3). \end{array}\right. \tag{4}$$

Now, substituting (4) in (1) after simple trigonometric pre-education, we get

$$\begin{cases} x = R_3\cos(\varphi_1+\varphi_2+\varphi_3) + l_3\cos(\varphi_1+\varphi_2) + l_2\cos\varphi_1 \\ y = R_3\sin(\varphi_1+\varphi_2+\varphi_3) + l_3\sin(\varphi_1+\varphi_2) + l_2\sin\varphi_1. \end{cases} \tag{5}$$

FORMULATION OF THE OPTIMIZATION PROBLEM

At present, the control system of the nuclear fuel reloading machine at the BN-800 reactor is operating in sequential mode, i.e. only one plug rotates at a time. Moreover, for some areas they used the by rotation only two plugs. Note that in this case, you can use the optimal algorithm proposed in [4, 5] for simultaneous control of two rotating plugs. The technical capabilities of the fuel overload mechanism allow the simultaneous control of several plugs. In this paper, we consider an algorithm for sequential control of plugs, which minimizes the time it takes to complete a move from one position to another. It should be noted that the problem of controlling of plugs of reactor is related to the task of controlling robotic systems [6, 7].

During the first rotation of a large cork around a fixed axis passing through the point O_1 at an angle $\Delta\varphi_1$, an average cork is rigidly fixed in its notch, and The system of three plugs (large, medium and small) rotates around a fixed axis as one absolutely solid body. We assume that the controlling moment u_1, $|u_1| \le \mu_1$ acts on this system. The motion of the system is described by the equation.

$$\hat{J}_1\ddot{\varphi}_1 = u_1. \tag{6}$$

Here $\hat{J}_1$ is the moment of inertia of the system of three plugs (large, medium and small) about the axis passing through the point O_1. The system can be transferred from the initial position φ_{10} to the position determined by the angle $\varphi_{10} + \Delta\varphi_1$ according to [4] during the time

$$\vartheta_1 = 2\sqrt{\frac{\hat{J}_1}{\mu_1}|\Delta\varphi_1|}. \tag{7}$$

At the second turn, the big plug jam is fixed. The middle plug rotates around a fixed axis passing through the point O_2 at an angle $\Delta\varphi_2$ a small plug is rigidly fixed in its notch, i.e. the system of two plugs (medium and small) rotates around a fixed axis as one absolutely solid body. We assume that the control moment acts on this system u_2, $|u_2| \le \mu_2$. The motion of the system is described by the equation

$$\hat{J}_2\ddot{\varphi}_2 = u_2, \tag{8}$$

where $\hat{J}_2$ is the moment of inertia of the system of two plugs (medium and small) about the axis passing through the point O_2. The system can be transferred from the initial position $\varphi_2 = \varphi_{20}$ to the position determined by the angle $\varphi_{20} + \Delta\varphi_2$ during the time

$$\vartheta_2 = 2\sqrt{\frac{\hat{J}_2}{\mu_2}|\Delta\varphi_2|}. \tag{9}$$

At the third turn large and medium plugs are fixed. The small plug rotates around a fixed axis passing through the point O_3 at an angle $\varphi_3 = \varphi_{30}$. We assume that the control moment u_3, $|u_3| \le \mu_3$. The motion of the system is described by the equation

$$\hat{J}_3\ddot{\varphi}_3 = u_3, \tag{10}$$

Here $\hat{J}_3$ is the moment of inertia of the small plug relative to the axis passing through the point O_3. The system can be transferred from the initial position $\varphi_3 = \varphi_{30}$ to the position determined by the angle $\varphi_{30} + \Delta\varphi_3$ during the time

$$\vartheta_3 = 2\sqrt{\frac{\hat{J}_3}{\mu_3}|\Delta\varphi_3|}. \tag{11}$$

According to (7), (9) and (11) the sequential transfer of the considered mechanical system from the initial position to the final position occurs during

$$\vartheta = \vartheta_1 + \vartheta_2 + \vartheta_3 = 2\left(\sqrt{\frac{\hat{J}_1}{\mu_1}|\Delta\varphi_1|} + \sqrt{\frac{\hat{J}_2}{\mu_2}|\Delta\varphi_2|} + \sqrt{\frac{\hat{J}_3}{\mu_3}|\Delta\varphi_3|}\right). \tag{12}$$

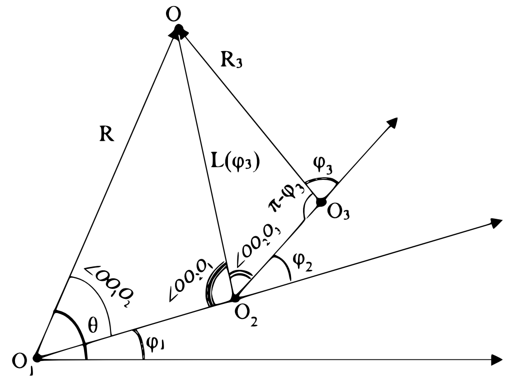

FIGURE 2. The angles φ_1, φ_2 and φ_3.

We will consider the problem of minimizing the functional (12) with respect to the variable $\Delta\varphi_1,\ \Delta\varphi_2$ and $\Delta\varphi_3$. In this case, the angles $\Delta\varphi_1,\ \Delta\varphi_2$ and $\Delta\varphi_3$ according to (5) must satisfy the constraints

$$\begin{cases} x = R_3\cos(\varphi_{10}+\varphi_{20}+\varphi_{30}+\Delta\varphi_1+\Delta\varphi_2+\Delta\varphi_3) \\ \quad +l_3\cos(\varphi_{10}+\varphi_{20}+\Delta\varphi_1+\Delta\varphi_2)+l_2\cos(\varphi_{10}+\Delta\varphi_1) \\ y = R_3\sin(\varphi_{10}+\varphi_{20}+\varphi_{30}+\Delta\varphi_1+\Delta\varphi_2+\Delta\varphi_3) \\ \quad +l_3\sin(\varphi_{10}+\varphi_{20}+\Delta\varphi_1+\Delta\varphi_2)+l_2\sin(\varphi_{10}+\Delta\varphi_1 \end{cases} \tag{13}$$

and additional constrains $|\varphi_i| \le \pi,\ \ i = 1,2,3.$

SOLVING OF THE OPTIMIZATION PROBLEM

The problem of minimizing the functional (12) with constraints (13) is the problem of nonlinear programming. The functional is non-differentiable, and the convexity property is also absent. Three variables are related by two conditions (13). Thus, in fact, one variable is independent. However, we cannot analytically express two of these variables in terms of the third.

We will express the angles $\varphi_1 = \varphi_{10} + \Delta\varphi_1$ and $\varphi_2 = \varphi_{20} + \Delta\varphi_2$ through the angle $\Delta\varphi_3$ ($\varphi_3 = \varphi_{30} + \Delta\varphi_3$), subject to the constraint (13), from geometrical considerations. Consider the triangles OO_1O_2 and OO_2O_3 (see Fig. 2). From the picture we see that $\varphi_2 = \varphi_{20} + \Delta\varphi_2 = \pi - \angle OO_2O_3 - \angle OO_2O_1$.

From the cosine theorem of triangles OO_2O_3 we nave

$$L^2(\Delta\varphi_3) = R_3^2 + l_3^2 + 2R_3l_3\cos(\varphi_{30} + \Delta\varphi_3).$$

Using the cosine theorem of triangles OO_1O_2 and OO_2O_3 , we get

$$\cos\angle OO_2O_3 = \frac{L^2(\Delta\varphi_3) + l_3^2 - R_3^2}{2l_3L(\Delta\varphi_3)},$$

$$\cos\angle OO_2O_1 = \frac{L^2(\Delta\varphi_3) + l_2^2 - R^2}{2l_2L(\Delta\varphi_3)}.$$

The dependence of the angle φ_1 on φ_3 we can find from the equality (see. Fig. 2)

$$\varphi_1 = \varphi_1 = \varphi_{10} + \Delta\varphi_1 = \theta - \angle OO_1O_2$$

and $\tan\theta = \frac{y}{x}$,

$$\cos\angle OO_1O_2 = \frac{R^2 + l_2^2 - L^2(\Delta\varphi_3)}{2Rl_2}$$

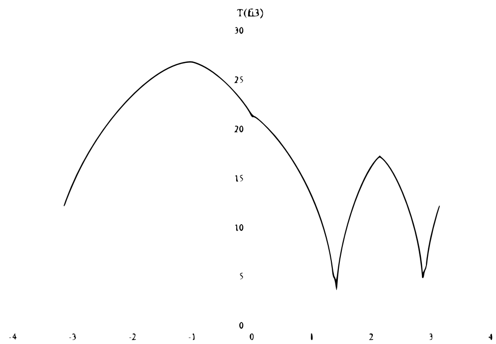

FIGURE 3. x =0,5; y = 0,5; φ_{10} = 0, φ_{20} = 0 φ_{30} = 0,1.

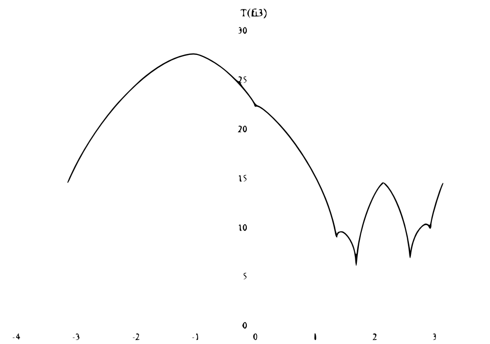

FIGURE 4. x =0,6; y = 0,4; φ_{10} = 0, φ_{20} = 0 φ_{30} = 1.

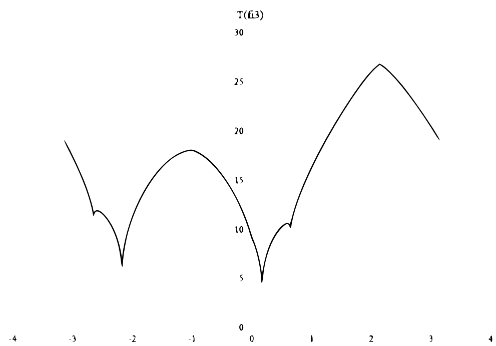

FIGURE 5.x =0,5; y = 0,5; φ_{10} = -1, φ_{20} = 1 φ_{30} = 1.

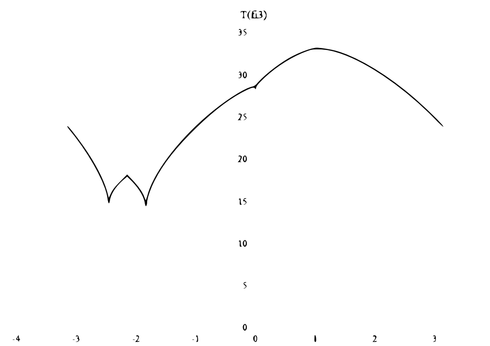

FIGURE 6. x =0,45; y = 0,55; φ_{10} = 0, φ_{20} = 0 φ_{30} = 1.

Below, we present graphs of the dependences of the time of transfer of the fuel overload mechanism from one position to another as a function of φ_3 at various initial and final positions. From these graphs, we see that function $\vartheta(\varphi_3)$ is not convex, and is also not differentiable at points of extremum. In addition, it is a multi-extremal function. Note that the time required for graphing is insignificant. Therefore, in this problem, the construction of the graph of the function and the search for a global extremum is quite acceptable.

CONCLUSIONS

The paper proposes an optimal algorithm for controlling the nuclear fuel overload mechanism, which allows transferring the overload mechanism from one position to another in the shortest time with the sequential method of controlling the plugs on the reactor cover.

ACKNOWLEDGMENTS

The research was supported by the Integrated Scientific Program of the Ural Branch of Russian Academy of Sciences, Project 18-1-1-9 “Estimation of the Dynamics of Nonlinear Control Systems and Route Optimization”.

REFERENCES

[1] V. V. Korobrin, A. N. Sesekin, O. L. Tashlykov, A. G. Chentsov. *Routing methods and their applications in improving the safety and efficiency of operation of nuclear power plants.* New technologies, Moscow (2012) A. I. Beltyukov, S. A.

[2] Karpenko, S. A. Poluyaktov, O. L. Tashlykov, G. P. Titov, A. M. Tuchkov, S. E. Schek-lein. *Nuclear power plants with fast neutron reactors with sodium coolant: a training manual.* Part 1. Eka-terinburg: UrFU, (2013).

[3] R. A. Becker. *Introduction to theoretical mechanics.* WcGrao-Hill. (1954).

[4] Yu. F. Dolgii, A. N. Sesekin, O. L. Tashlykov, E. Z. Zaynullina. *Optimal control of the fuel reload mechanism.* IFAC - PaperOnLine vol. 51, No. 32, pp. 636–641. (2018).

[5] Yu. F. Dolgii, A. A. Petunin, A. N. Sesekin, O. L. Tashlykov. *Optimal control of the system of coupled cylinders.* AIP. Conference Proceeding vol. 2048, 020007 (1-6) (2018).

[6] F. L. Chernoushko, N. N. Bolotnik, V. G. Hradecky. *Manipulation robots.* Moscow: Nauka (1989).

[7] M. W. Spong, S. A. Hutchinson, M. Vidyasagar. *Robot Modeling and Control.* John Wiley and Sons (2006).